SOLVING QUADRATIC INEQUALITIES IN ONE VARIABLE BY SIGN ANALYSIS

A WORKBOOK FOR GRADE 9 - MATHEMATICS

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Solving Quadratic Inequalities in one variable by SIGN ANALYSIS: A PREFACE

Dear Parents, Teachers, and Students,

The COVID-19 pandemic is indeed a test of many things – especially to our patience, resiliency, and motivation to get through this very difficult situation. However, we should not let ourselves succumb to the looming consequences of this adversity.

Due to the undesirable effects of the current pandemic to the education sector, the Far Eastern University – Institute of Education is committed to bring response to the restrictions brought by this pandemic. Spearheaded by its esteemed faculty members, FEU – IE has come up to an idea in creating modules and workbooks across levels that will give advantage to the students especially those who do not have access to the internet, and those who are experiencing struggles vis-à-vis the conduct of online classes. As face-to-face meetings are still restricted due to the aggravating pandemic situation, FEU-IE wants our young learners to continue learning under the given circumstance, which led to the creation of FEU-IE Basic Education Learning Packets.

We have to understand that learning must go on amidst these uncertain times provided that there is proper support and guidance given by "more-knowledgeable-others". This workbook is designed to bring out simple ideas which can be easily grasped by our students. Supplemented with guided discussion and practice exercises, this workbook for Grade 9 – Mathematics will aid students to clearly understand the core concepts regarding Quadratic Inequalities.

- G. Baybayon

What you need to know . . .

LEARNING OBJECTIVES

As you go through this workbook, you are expected to:

- Recall the factors of the quadratic expression
- Identify the intervals of a quadratic inequality on a number line
- Graph and solve quadratic inequalities in one variable by sign analysis; and
- Develop the "value of acceptance" as the lesson about solving quadratic inequalities reveals that there are things that are not meant to be equal especially in real-life situations

CONTENT

This workbook focuses on Quadratic Inequalities specifically by looking at its solutions by utilizing the "Sign Analysis".

Prerequisite Concept: Linear Inequalities, Quadratic Equations

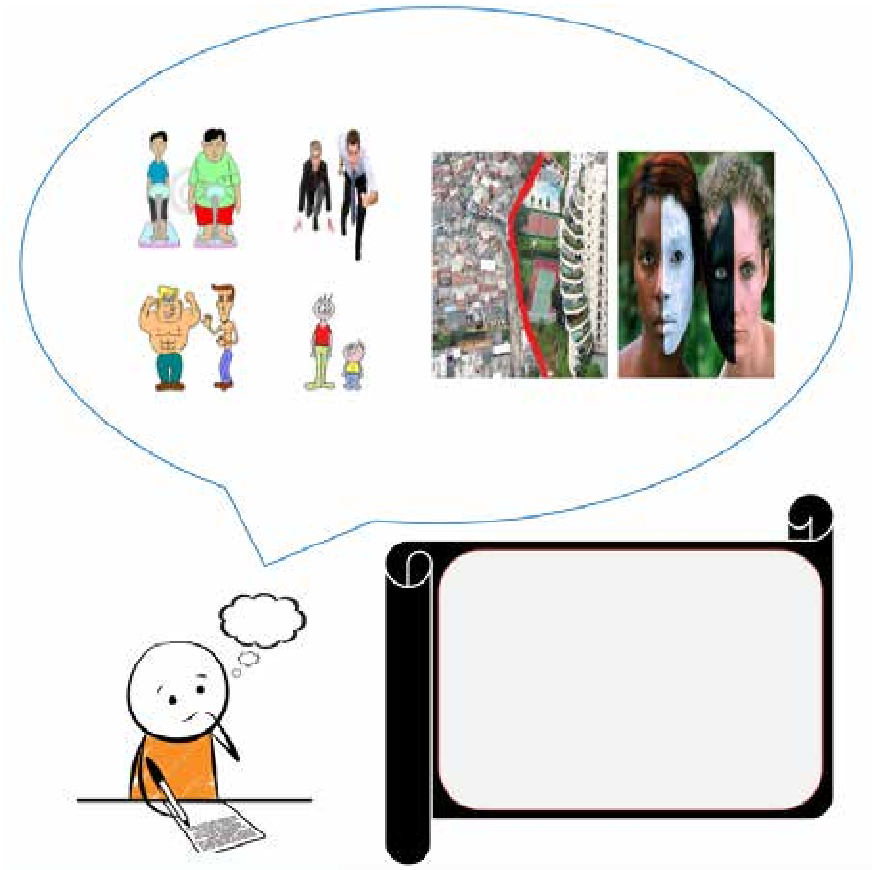
REFERENCES

- Bryant, Merden L., Bulalayao, Leonides E., et. al., Mathematics Grade 9 Learner's Material

LET'S GET STARTED

ACTIVITY 1: CAN YOU SPOT THE DIFFERENCE?

Look closely at these pictures. What can you observe? Provide your insights in the box provided.



The pictures will be differentiated in terms of:

- Weight
- Height
- Color
- Body Mass
- Age

TEACHER'S NOTE:

Actually, all things are not created equal - maybe in appearance, quantity, observable features or sometimes, when it comes to quality. It is a clear fact that you can't compare two things that are entirely different. But then, it is only applicable to us, humans. It is true that we are all created equal and for that reason, it is also implied that people should not discriminate and hence, they do not have the license to judge one's imperfections.

NOW, IT'S YOUR TURN . . .

Realizations . . .

What did I learn from this activity? What are the lessons that I can share with my fellow students? Write your insights in the space provided.

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GETTING READY . . .

There is such thing as "unequal" in Mathematics. When we are dealing with equations, we say that the left and right side of the equation hold the same value. But what if it happens that one side is greater or lesser than the other?

That concept has something to do with *inequality*.

LET’S HAVE SOME DRILL – UNLOCKING DIFFICULTIES

ACTIVITY 2: “QUADRATIC EQUATIONS OR NOT?”

Direction:

Identify each of the following mathematical statements if it is a quadratic equation or not. Write your answers in the space provided.

1. $x^2 + 3x - 10 > 0$ ________________________________
2. $3m^2 - 7m - 10 = 0$ ________________________________
3. $r^2 + 2r - 35 \leq 0$ ________________________________
4. $4b^2 + 28m + 276 = 0$ ________________________________
5. $-\frac{4}{3} c^2 = 256$ ________________________________

ANALYSIS:

Considering these mathematical sentences above, which are quadratic equations, and which are not? How do you describe then a quadratic equation?

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ACTIVITY 3: “CAN YOU FACTOR ME?”

Direction: Factor each of the following polynomials. Write your answers in the space provided.

1. $-3s^2 + 9s$ ________________________________
2. $4m + 24m^2$ ________________________________
3. $15r^2 + 10r - 35$ ________________________________
4. $b^2 + 8y + 12$ ________________________________
5. $x^2 - 10x + 21$ ________________________________
6. $4c^2 + 20c + 25$ ________________________________
7. $a^2 - 4a + 4$ ________________________________
8. $x^2 - 4$ ________________________________

ANALYSIS:

How did you factor each polynomial and what were the possible techniques that you used to come up with the right factors?

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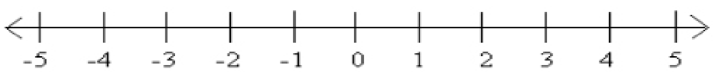
ACTIVITY 4:

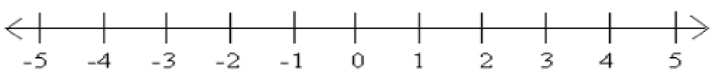
Direction:

Graph the given linear inequalities in the provided number line.

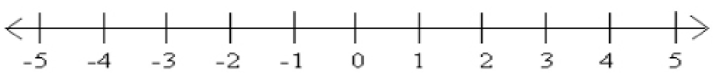
a. $x < -3$



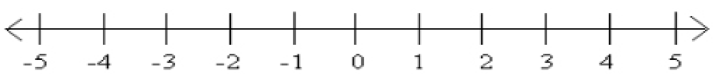
b. $x \geq 2$



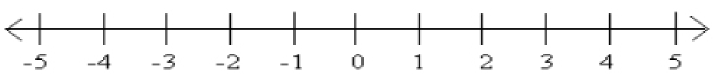
c. $x > -1$



d. $x - 3 \leq -1$



e. $2x - 5 < -3$



TEACHER'S NOTE:

It is very important for you to know the process on how to graph linear inequalities in one variable. (e.g., when to use the open and shaded circles, and the parentheses and brackets in writing for the solution set).

STUDENT'S CORNER:

Did you encounter any difficulty from the activity? Write your thoughts here.

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TEACHER'S NOTE:

Recall that . . .

A quadratic equation in one variable is a mathematical sentence of degree 2 that can be written in the following standard form; $ax^2 + bx + c = 0$, where a, b and c are real numbers and $a \neq 0$.

Hence, looking at items 1 and 3 in Activity 2, it can be inferred that the left side of the equation may be less than, "less than or equal to" or it may happen that it is greater than, "greater than or equal to" the right side of the equation and vice-versa.

CONCEPTUAL DEVELOPMENT:

“QUADRATIC INEQUALITIES”

A quadratic inequality in one variable is an inequality that contains a polynomial of degree 2 and can be written in any of the following forms:

i. $ax^2 + bx + c > 0$

ii. $ax^2 + bx + c < 0$

iii. $ax^2 + bx + c \geq 0$

iv. $ax^2 + bx + c \leq 0$

where a, b and c are real numbers and $a \neq 0$.

“QUADRATIC INEQUALITIES”

For now, we will be dealing with a unique method on how to solve quadratic inequalities in one variable and that is by using sign analysis.

To solve a quadratic inequality in one variable by using sign analysis, the following procedures can be followed:

1. *First, we will find for the roots of its corresponding quadratic equation by using any of the four methods on how to solve quadratic equations (By Extracting the Square Roots, Factoring, Completing the Square or by Quadratic Formula)*

2. *The roots of the corresponding equation will serve as the critical values.*

3. *When the critical values are plotted on the number line, it will separate the number line into two or three regions. Each region corresponds to a specific interval. An interval is a part of the solution of an inequality if a number in that interval makes the inequality true. By using sign analysis, we will know if the given interval is included in the solution or not, depending on the inequality symbol.*

Note: For the in-depth discussion regarding the steps on how to solve quadratic inequalities, refer to examples 1 and 2 below.

Note: Before you proceed, you should have mastered the plotting of critical values on the number line and also the process on how to solve quadratic equations (by extracting the square roots, by factoring, by completing the square, and by Quadratic Formula) based on our previous exercises. Which of the following methods of solving quadratic equations you find difficult to follow? Write your insights here.

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Example # 1:

Find the solution set of $x^2 + 3x - 10 > 0$ then graph.

1. First, let us solve for the roots of the given inequality by using its corresponding quadratic equation.

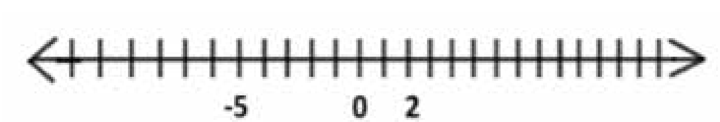
$x^2 + 3x - 10 = 0$ (*Corresponding quadratic equation*)

$(x + 5)(x - 2) = 0$ (*Factor the left – side of the equation*)

$(x + 5) = 0$ and $(x - 2) = 0$ (*Equate both factors to zero*)

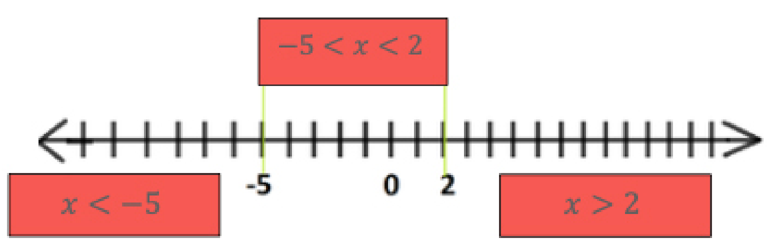
$x_1 = -5$ *and* $x_2 = 2$ *(Solve for the values of x)*

2. Plot the points corresponding to -5 and 2.



The three intervals are:

- $x < -5$ or $-\infty < x < -5$
- $-5 < x < 2$
- $x > 2$ or $2 < x < +\infty$



3. Now, we will use the sign analysis to determine the solution set of the given inequality.

Construct a table which includes all the factors for the 1st part and the intervals (which contains the 1st interval, the 2nd interval and the critical values in between the former, and the last interval) & test values for the 2nd part. By using our table of signs, we will list all the factors and also the intervals (Refer to the given figure below).

FACTORS	INTERVALS				
	$x < -5$	x=-5	$-5 < x < 2$	x=2	$x > 2$
	Test Values: -7	-5	0	2	5
$(x+5)$	–	0	+	+	+
$(x-2)$	–	–	–	0	+
$(x+5)(x-2)$	+	0	–	0	+

We will then choose a number that is within the given interval. Those numbers will serve as our test values. The test values will be substituted to each of the factors under its interval and we will check if the obtained value is positive or negative. Take note that we are not after for the exact value or resulting number but instead, we will just be focusing on its sign.

Let us analyze the table . . .

- Considering our table of signs, we can pick any number (under the row of test values) within the given interval. Do the same with the other intervals.
- In the first interval, we chose −7 under that interval which is $x < -5$ or $-\infty < x < -5$, as our test value that will be substituted to the given factors. After substituting −7 to the first factor which is (x + 5) and to the second factor which is (x − 2), we found out that the values are both negatively signed numbers. Since we have to multiply the given factors (x+ 5) and (x − 2) because this will yield to the left side of our quadratic inequality, we also have to multiply the signs. Under this condition, negative value (−) multiplied with a negative value (−) is always positive (+).

- We also tested for the first critical value itself which is −5. Substituting this to the factors, we obtained 0 and (–), respectively. We know that the product of the two is always equal to 0.
- For the second interval $-5 < x < 2$, we chose 0 as our test value. Substituting this to our given factors, we will have (+) and (–) values, respectively. Notice that we will have a (–) result after multiplying the two values.
- We also tested for the second critical value itself which is 2. Substituting this to the given factors then we will have (+) and 0, respectively. Still, we know that the product of these is equal to 0.
- Now, for our third interval which is $x > 2$ or $2< x < +\infty$, we chose 5 as our test value. Substituting this to our given factors, we will have (+) for our first factor and also (+) for our second factor. We know that (+) multiplied with a (+) value is always equal to (+).

Going back to our original quadratic inequality which is $x^2 + 3x - 10 > 0$ and which is also equal to $(x+5)(x-2)>0$, what must be the value of the left side of the inequality so that it will be greater than 0?

Now, we ask this question: By looking at our table of signs, what interval has the (+) value numbers?

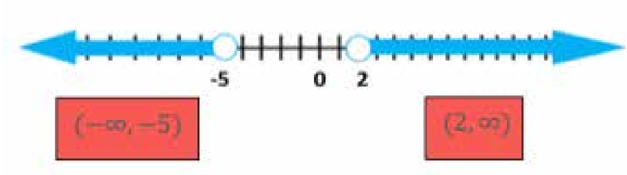
It can be inferred that the left side of the inequality must be positive so that it will be greater than 0.

Therefore, the inequality $x^2 + 3x - 10 > 0$ is true for any of the value of x in the interval,

$$x <-5 \text{ or } -\infty < x <-5 \text{ and}$$
$$x >2 \text{ or } 2< x <+\infty,$$

And these intervals exclude -5 and 2.

The **SOLUTION SET** of the inequality is $(-\infty, -5) \cup (2, \infty)$ and its graph is shown below:



Example # 2:

Find the solution set of $x^2 + 2x - 35 \leq 0$ then graph.

1. First, let us solve for the roots of the given inequality by using its corresponding quadratic equation.

$x^2 + 2x - 35 = 0$ (*Corresponding quadratic equation*)

$(x + 7)(x - 5) = 0$ (*factor the left-side of the equation*)

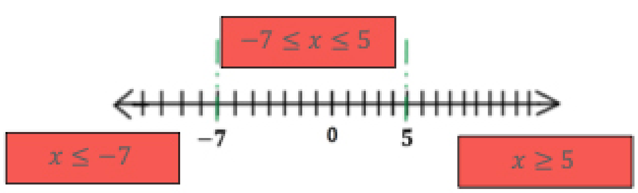
$(x + 7) = 0$ and $(x - 5) = 0$ (*Equate both factos to zero*)

$x_1 = -7$ and $x_2 = 5$ (*Solve for the values of x*)

2. Plot the points corresponding to -7 and 5.

The three intervals are:

- $x \leq -7$ or $-\infty < x \leq -7$
- $-7 \leq x \leq 5$
- $x \geq 5$ or $5 \leq x < +\infty$



3. Now, we will use the sign analysis to determine the solution set of the given inequality.

Construct a table which includes all the ***factors*** for the 1st part and the ***intervals*** (which contains the 1st interval, the 2nd interval and the critical values in between the former, and the last interval) & ***test values*** for the 2nd part. By using our table of signs, we will list all the factors and also the intervals (Refer to the given figure below).

FACTORS	INTERVALS				
	$x < -7$	**x=-7**	$-7 < x < 5$	**x=5**	$x > 5$
	Test Values: -10	-7	0	5	10
$(x+7)$	–	0	+	+	+
$(x-5)$	–	–	–	0	+
$(x+7)(x-5)$	+	0	–	0	+

We will then choose a number that is within the given interval. Those numbers will serve as our test values. The test values will be substituted to each of the factors under its interval and we will check if the obtained value is positive or negative. Take note that we are not after for the exact value or resulting number but instead, we will just be focusing on its sign.

Let us analyze the table . . .

- Considering our table of signs, we can pick any number (under the row of test values) within the given interval. Do the same with the other intervals.
- In the first interval, we chose −10 under x < −7 as our test value that will be substituted to the given factors. After substituting −10 to the first factor which is (x + 7) and to the second factor which is (x − 5), we found out that the values are both negatively signed numbers. Since we have to multiply the given factors (x + 7) and (x − 5) because this will yield to the left side of our quadratic inequality, we also have to multiply the signs. Under this condition, negative value (−) multiplied with a negative value (−) is always positive (+).

- We also tested for the first critical value itself which is −7. Substituting this to the factors, we obtained 0 and (−), respectively. We know that the product of the two is always equal to 0.
- For the next interval $-7 < x < 5$, we chose 0 as our test value. Substituting this to our given factors, we will have (+) and (−) values, respectively. Notice that we will have a (−) result after multiplying the two values.
- We also tested for the second critical value itself which is 5. Substituting this to the given factors then we will have (+) and 0, respectively. Still, we know that the product of these is equal to 0.
- Now, for our last interval which is $x > 5$ or $5 < x < +\infty$, we chose 10 as our test value. Substituting this to our given factors, we will have (+) for our first factor and also (+) for our second factor. We know that (+) multiplied with a (+) value is always equal to (+).

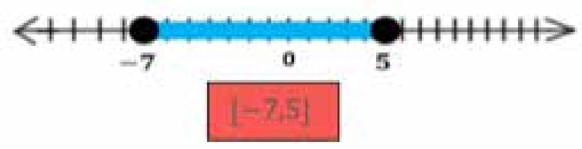
Going back to our original quadratic inequality which is $x^2 + 2x - 35 \leq 0$ and which is also equal to $(x+7)(x-5) \leq 0$, what must be the value of the left side of the inequality so that it will be less than or equal to 0?

Now, we ask this question: By looking at our table of signs, what interval has the (−) value numbers and 0?

It can be inferred that the left side of the inequality must be a number which is less than 0 or equal to 0 so that it will satisfy the inequality condition.

Therefore, the inequality $x^2 + 2x - 35 \leq 0$ is true for any of the value of x in the interval, $-7 \leq x \leq 5$. And this obtained interval includes -7 and 5.

The **SOLUTION SET** of the inequality is [−7,5] and its graph is shown below:



STUDENT'S CORNER:

Did you encounter any difficulty from the discussion and examples given? Write your thoughts here.

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Now, it's your turn . . .

PRACTICE EXERCISES:

ACTIVITY 5: "Complete Me, Graph Me"

Direction: Complete the table of signs and find the solution set of the given quadratic inequalities then graph. Show your complete solution.

1. $x^2 - 4x - 12 > 0$

FACTORS	INTERVALS				
	Test Values:				

SOLUTION SET:

2. $x^2 + x - 20 \leq 0$

FACTORS	INTERVALS				
	Test Values:				

SOLUTION SET:

3. $x^2 + 5x - 14 \geq 0$

FACTORS	INTERVALS				
	Test Values:				

SOLUTION SET:

4. $x^2 + 8x + 16 > 0$

FACTORS	INTERVALS				
	Test Values:				

SOLUTION SET:

5. $x^2 + 2x + 3 < 0$

FACTORS	INTERVALS				
	Test Values:				

SOLUTION SET:

TEACHER'S NOTE:

In summary . . .

To solve a quadratic inequality in one variable by using sign analysis, the following procedures can be followed:

- First, we will find for the roots of its corresponding quadratic equation by using any of the four methods on how to solve quadratic equations (By Extracting the Square Roots, Factoring, Completing the Square or by Quadratic Formula)

- The roots of the corresponding equation will serve as the critical values.

- When the critical values are plotted on the number line, it will separate the number line into two or three regions, and each region corresponds to a specific interval. An interval is a part of the solution of an inequality if a number in that interval makes the inequality true. By using sign analysis, we will know if the given interval is included in the solution or not depending on the inequality symbol.

STUDENT'S CORNER:

What can you say about this lesson regarding Quadratic Inequalities? Share your insights in the space provided.

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