Republic of the Philippines
OFFICE OF THE PRESIDENT
COMMISSION ON HIGHER EDUCATION

CHED MEMORANDUM ORDER (CMO)
No. 10
Series of 2011

SUBJECT: POLICIES AND STANDARDS FOR THE MASTER OF SCIENCE IN MATHEMATICS (M.Sc. MATHEMATICS) PROGRAM

In accordance with the pertinent provisions of Republic Act (RA) No. 7722, otherwise known as the "Higher Education Act of 1994", by virtue of Commission en banc **Resolution No. 060-2011** dated **05 April 2011**, and for the purpose of rationalizing the graduate mathematics education in the country with the end view of keeping apace with the demands of global competitiveness, the following Policies and Standards are hereby adopted and promulgated by the Commission.

ARTICLE I
INTRODUCTION

Graduate level study is the advanced preparation and mastery of a specialized field or discipline. It occurs within an atmosphere of intellectual and creative rigor that encourages scholarly inquiry, research and study of the evolving formulation of knowledge. Graduate education results in the candidate's ability to shape the direction of one's discipline, to become a leader in one's respective profession, and to contribute to the rapidly changing global community.

ARTICLE II
AUTHORITY TO OPERATE

All private higher education institutions (HEIs) intending to offer the degree stated above must secure proper authority from the Commission in accordance with existing rules and regulations. State Universities and Colleges (SUCs), and Local Universities and Colleges (LUCs) should likewise strictly adhere to the provisions stated in these policies and standards.

ARTICLE III
PROGRAM SPECIFICATIONS

Section 1. Degree Name

The degree program herein shall be called Master of Science in Mathematics.

Section 2. Objectives of the Program

The M.Sc. Mathematics program is designed to achieve the following objectives:

1. To provide students with a foundation sufficient to pursue careers in academe, industry and government requiring mathematical skills and perspectives;
2. To prepare the student for doctoral studies in pure or applied mathematics, statistics or some related field;
3. To provide foundations and initial training for doing research in mathematics; and
4. To enhance competence in teaching senior level undergraduate mathematics courses.

ARTICLE IV
COMPETENCY STANDARDS

Graduates of the M.Sc. Mathematics program are expected to:

1. Demonstrate success in teaching and possess the ability to work with undergraduates in research.
2. Achieve a certain level of fluency in advanced mathematics.
3. Be able to pursue various career options such as research, teaching and other professional practice.

ARTICLE V
CURRICULUM

Section 3. Curriculum Description

Two aspects of modern developments in mathematics are reflected in the design of the graduate program: its internal structure marked by deep interconnection between its various branches, and its interdisciplinary nature–traditionally with physics and engineering, but now extending to other fields. The required courses provide substantial background in the traditional foundational areas of mathematics. Mastery of a mathematical specialty is enhanced by a good choice of specialization and elective courses.

The major external influence on mathematics is the computer and everything related to domain will be of central importance to mathematics in the coming century. A good graduate program must reflect these developments in scientific computing and information technology by providing opportunities for students to venture into cutting-edge areas requiring advanced mathematics such as Mathematical Finance, Mathematical Biology, and Mathematical Modeling for the Life and Physical Sciences. HEIs are therefore encouraged to develop competent faculty and researchers in these niche areas.

The curriculum for the M.Sc. Mathematics program should contain a total of at least 33 units, broken down into 15 units of Required Courses, 12 units of Electives and 6 units of Master's Thesis.

The electives are specialized courses in pure and applied mathematics. Since the mathematics departments of different schools will have their particular strengths and orientation, the elective courses will allow for flexibility and accommodate the special interests of the various departments. In addition to the 27 units of course work, the department may offer courses in mathematics education, and/or methods of research.

Section 4. Curriculum Outline

The minimum requirements for the M.Sc. in Mathematics are outlined in Table 1.

Table 1. Components of the M.Sc. Mathematics Curriculum and the Corresponding Units for the Courses

COMPONENT	UNITS
Required Courses	15
Electives	12
Master's Thesis	6
Total	**33**

Section 5. Required Courses

The Required courses for M.Sc. Mathematics are outlined in Table 2.

Table 2. List of Required Courses for the M.Sc. Mathematics Program

DESCRIPTIVE TITLE	UNITS
1. Abstract Algebra I	3
2. Complex Analysis	3
3. Linear Algebra	3
4. Real Analysis I	3
5. Topology or Geometry Course	3
Total	**15**

Section 6. Sample Electives

Table 3. Sample Electives for the M.Sc. Mathematics Program

DESCRIPTIVE TITLE	UNITS
Abstract Algebra II	3
Algebraic Geometry	3
Algebraic Number Theory	3
Algebraic Topology	3
Approximation Theory	3
Coding Theory	3
Combinatorial Mathematics	3
Design Theory	3
Differential Geometry	3
Functional Analysis	3
Geometric Crystallography	3
Graph Theory	3
Group Theory	3
Hyperbolic Geometry	3
Lie Algebra	3
Mathematics in Population Biology	3

DESCRIPTIVE TITLE	UNITS
Mathematical Finance	3
Mathematical Statistics	3
Multivariate Analysis	3
Numerical Analysis I	3
Numerical Analysis II	3
Numerical Optimization	3
Probability Theory	3
Projective Geometry	3
Real Analysis II	3
Stochastic Calculus	3
Theory of Ordinary Differential Equations	3
Theory of Partial Differential Equations	3

HEIs may offer electives other than those specified in the above list, according to their faculty and institutional resources and thrusts.

Section 7. Master's Thesis

The master's thesis must be a scholarly contribution to mathematical knowledge. The procedure to be followed in complying with this requirement shall be specified by the Department consistent with relevant standard operating procedures of the HEI and in accordance with pertinent CHED memoranda.

The student must present the results of the thesis research in a seminar/lecture. Publication of the master's thesis is strongly encouraged.

Section 8. Comprehensive Examination

The Comprehensive Examination is a written examination that the student shall take after she/he has passed all the Required Courses. It is intended to determine whether the student has sufficient broad mathematical knowledge. It shall cover three areas: Algebra, Analysis and a third one to be chosen by the student.

The Department shall administer the exam and shall set the requirements for passing and retakes. A student who passes the Comprehensive Examination is advanced to candidacy for the M.Sc. in Mathematics degree.

Section 9. Recommended Program of Study

Table 4 gives the recommended program of study. HEIs may adhere to this, or when necessary, modify the sequencing of courses.

Table 4. Recommended Sequence of Courses in the M.Sc. Mathematics Program

1st Year / 1st Semester	Units	1st Year / 2nd Semester	Units
Abstract Algebra I	3	Real Analysis I	3
Linear Algebra	3	Geometry or Topology	3
Complex Analysis	3	Elective II	3
Elective I	3	Elective III	3
Total	**12**	***Total***	**12**

2nd Year / 1st Semester		2nd Year / 2nd Semester	
Elective IV Comprehensive Examination	3	Master's Thesis	3
Total	**3**	***Total***	3
2nd Year Summer			
Master's Thesis			3
		TOTAL	*33*

ARTICLE VI
LEARNING RESOURCES AND SUPPORT

PROGRAM ADMINISTRATION

Section 10. Qualifications of the Unit Head

The minimum qualifications of the unit head that implements the degree program are the following:

10.1Dean of the Unit/College

The dean of a unit/college must be at least a master's degree holder in any of the disciplines for which the unit/college offers a program; and a holder of a valid certificate of registration and professional license, where applicable.

10.2Head of the Unit/Department

The head of the unit/department must be at least a master's degree holder in the discipline for which the unit/department offers a program.

Section 11. Administration of the Master's Program in Mathematics

The Department shall appoint a Graduate Committee (GC) that will be responsible for administering the graduate program in mathematics. The responsibilities of the GC include the following:

1. Set and implement policies on faculty, course offerings and research standards of the programs in accordance with pertinent CHED memoranda;
2. Review and monitor the progress of students in the program; and
3. Attend to all developmental needs of the program.

Section 12. Institutional Responsibility

Through the Graduate Committee, the institution shall see to it that the student completes the program of study on time and that the program maintains the expected academic standard.

Section 13. Faculty Advising

A thesis adviser should be a holder of a Ph.D. degree in Mathematics or related fields, or a holder of M.Sc. degree in Mathematics or related fields with a track record in research and publication.

Faculty advising is a crucial component of the graduate program. Therefore, advising should be given a corresponding faculty load equivalent or a commensurate honorarium. The number of credit units or honorarium is left to each institution to determine.

Section 14. Faculty

14.1. Qualifications of faculty

A HEI that offers a graduate program in mathematics must have competent full-time faculty whose qualifications include not only graduate degrees in mathematics and/or related fields but also a good track record in research.

To offer a M.Sc. Mathematics program, the unit/department must satisfy the following minimum requirements:

1. The graduate faculty must consist of at least 5 faculty members teaching the program. The unit/department must have at least three (3) faculty members with an earned Ph.D. Mathematics degree. At least two (2) of these faculty members must have a full -time status with the institution.
2. The faculty is expected to be actively engaged in research and to publish in reputable scientific journals.
3. The faculty is encouraged to be active members of recognized professional organizations in mathematics.
4. The faculty is encouraged to be involved in extension activities.

Section 15. Library

HEIs with M.Sc. Mathematics program must meet the minimum requirements for the BS Mathematics program. In addition, the library must maintain a collection of the core and elective courses in the curriculum and constantly update and expand their collections. There must also be active subscriptions to mathematics journals either in print or online. Internet access is required for the masters program in mathematics.

Section 16. Laboratory and Facilities

The minimum laboratory requirements for the BS Mathematics program must be met. Class size may vary depending on the allowable number of students as may be feasible to economically offer a graduate class.

The HEI should provide the necessary infrastructure for research in mathematics, such as computer facilities, relevant and up-to-date mathematical and computing software.

Graduate students should have access to computer facilities equipped with relevant mathematical and computing software.

The HEI should also provide the necessary infrastructure for research in mathematics.

ADMISSION AND RETENTION REQUIREMENTS

Section 17. Admission and Retention

Admission in the M.Sc. program in mathematics shall require the following:

1. A bachelor's degree in mathematics or other related programs from a recognized institution of higher learning.
2. Satisfaction of other usual admission requirements prescribed by the HEI.

Section 18. Grade Requirement

The HEI, through the Graduate Committee, shall set the standards for the grade requirement for the student to stay in the M.Sc. program in mathematics.

Section 19. Rules on Residency

The Graduate Committee, in accordance with the general rules of the HEI regarding this matter, shall set residency rules.

ARTICLE VII
TRANSITORY, REPEALING AND EFFECTIVITY PROVISIONS

Section 20. Transitory Provision

HEIs that have been granted permit or recognition for M.Sc. Mathematics are required to fully comply with all the requirements in this CMO, within a non-extendable period of three (3) years after the date of its effectivity. State Universities and Colleges (SUCs) and Local Colleges and Universities (LCUs) shall also comply with the requirements herein set forth.

Section 21. Repealing Clause

All CHED issuances, rules and regulations or parts thereof, which are inconsistent with the provisions of this CMO, are hereby repealed.

Section 22. Effectivity Clause

This CMO shall take effect fifteen (15) days after its publication in the Official Gazette, or in two (2) newspapers of national circulation. This CMO shall be implemented beginning Academic Year 2012-2013.

Quezon City, Philippines, May 12, 2011.

For the Commission,

PATRICIA B. LICUANAN, Ph.D.
Chairperson

Annex A - Course Specifications

ANNEX A
COURSE SPECIFICATIONS

The Commission has determined the minimum content of the courses included in the M.Sc. Mathematics programs as provided in the course outlines below. Suggested references are also provided, the latest versions of which are recommended.

ABSTRACT ALGEBRA I

Course Description:

Groups and their elementary properties, subgroups, cyclic groups, normal subgroups and quotient groups, permutation groups, group homomorphisms and isomorphisms, group actions, Sylow theorems, direct products and sums, finitely generated abelian groups, rings and their elementary properties, subrings and ideals, integral domains and fields, principal ideal domains and unique factorization domains.

Course Credit: **3 units**

Course Outline:

1. Fundamentals of Group Theory
   1.1 Elementary Properties of Groups
   1.2 Subgroups

2. Special Classes of Groups
   2.1. Cyclic Groups
   2.2. Abelian Groups
   2.3. Symmetric, Alternating and Dihedral Groups

3. Normal Subgroups and Homomorphisms
   3.1. Cosets and the Theorem of Language
   3.2. Normal Subgroups
   3.3. Quotient Groups
   3.4. Group Homomorphisms
   3.5. Isomorphism Theorems

4. Further Topics in Group Theory
   4.1. Direct Sums and Direct Products
   4.2. Free Abelian Groups
   4.3. Finitely Generated Abelian Groups
   4.4. Group Actions and Sylow Theorems

5. Fundamentals of Ring Theory
   5.1. Rings and Subrings
   5.2. Ideals: Prime, Maximal, Principal
   5.3. Integral Domains and Fields
   5.4. Euclidean Domains and Principal Ideal Domains

5.5. Unique Factorization Domains
5.6. Quotient Rings

Suggested References:

1. T.W. Hungerford, *Algebra*, Springer
2. I.N. Herstein, *Topics in Algebra*
3. N. Jacobson, *Basic Algebra I and II*
4. Joseph J. Rotman, *The Theory of Groups*, Springer

ABSTRACT ALGEBRA II

Course Description:

Field extensions, algebraic extensions, normal and separable extensions, splitting fields, algebraically closed field, finite fields, Galois theory and its applications, cyclic, cyclotomic and radical extensions.

Course Credit: 3 units

Course Pre-Requisite: Abstract Algebra I

Course Outline:

1. Review of Rings
   - 1.1. Rings and Homomorphisms
   - 1.2. Ideals and Quotient Rings
   - 1.3. Field of Quotients of an Integral Domain
   - 1.4. Euclidean and Unique Factorization Domains
   - 1.5. Polynomial Rings

2. Extension Fields
   - 2.1. Field Extensions
   - 2.2. Algebraic and transcendental elements and extensions
   - 2.3. Ruler and Compass Constructions

3. Fundamental Theorem of Galois Theory
   - 3.1. The Galois group and Galois extensions
   - 3.2. The Fundamental Theorem of Galois theory

4. Splitting Fields, Algebraic Closure and Normality
   - 4.1. Splitting Fields
   - 4.2. Algebraic Closures
   - 4.3. Separable Extensions
   - 4.4. Normal Extensions
   - 4.5. Fundamental Theorem of Algebra
   - 4.6. The Galois Group of a Polynomial

5. Further Topics in Galois Theory
   - 5.1. Finite Fields
   - 5.2. Cyclic Extensions

5.3. Cyclotomic Extensions
5.4. Radical Extensions

Suggested References:

1. T.W. Hungerford, *Algebra*, Springer
2. N. Jacobson, *Basic Algebra I and II*
3. I. Stewart, *Galois Theory*
4. E. Artin, *Galois Theory*, Dover

ALGEBRAIC GEOMETRY

Course Description:

Commutative algebra, algebraic curves and surfaces, affine and projective spaces, Gröbner bases, elimination and extension theorems, Hilbert's Nullstellensatz, affine and projective varieties, computational techniques, introduction to elliptic curves

Course Credit: 3 units

Course Pre-Requisite: Abstract Algebra I, Linear Algebra

Course Outline:

1. Some commutative algebra (polynomial rings, ideals)
2. Affine and Projective Spaces
3. Algebraic curves and surfaces
4. Gröbner bases and Buchburger's algorithm
5. Elimination and extension theorems
6. Hilbert's Nullstellensatz
7. Affine and projective varieties, functions on a variety
8. Computational techniques
9. Further topics (introduction to elliptic curves, Riemann-Roch Theorem)

Suggested References:

1. Phillip A. Griffiths, *Introduction to Algebraic Curves*, American Mathematical Society, 1989.
2. W. Fulton, Algebraic Curves: An Introduction to Algebraic Geometry, Addison-Wesley, 1989.
3. F. Kirwan, *Complex Algebraic Curves*, Cambridge University Press, 1992.
4. H. Matsumura, Commutative Ring Theory, Cambridge Studies in Advanced Mathematics
5. Robin Hartshorne, *Algebraic Geometry*, Springer Graduate Texts in Math (GTM 52)
6. D. Cox, J. Little, and D. O'Shea, Ideals, Varieties and Algorithms, 3rd Ed., Springer, 2007.

ALGEBRAIC NUMBER THEORY

Course Description:

Primes, algebraic number fields, algebraic integers, basic class field theory.

Course Credit: 3 units

Course Pre-Requisite: Elementary Number Theory, Modern Algebra

Course Outline:

1. Some elementary number theory
2. Diophantine equations
3. Primes and the fundamental theory of arithmetic
4. Quadratic reciprocity
5. Unique factorization in R[x] and the Gaussian integer ring
6. Algebraic number fields and algebraic integer rings
7. Trace and norm of algebraic integers
8. Ideals and unique factorization of ideals in number fields
9. The ideal class group and class number of a number field
10. Quadratic fields and cyclotomic fields
11. Decomposition and ramification of primes
12. Fermat's last theorem for regular p rimes
13. Residue characters and reciprocity laws

Suggested References:

1. Ireland and Rosen, *A Classical Introduction to Modern Number Theory*, Springer- Verlag, Graduate Texts in Mathematics (GTM) 84
2. L. Washington, *Introduction to Cyclotomic Fields*, Springer-Verlag, GTM 83
3. P. Ribenboim, *The Theory of Algebraic Numbers*

ALGEBRAIC TOPOLOGY

Course Description:

Homotopy, fundamental group, singular homology, simplicial complexes, degree and fixed-point theorems

Course Credit: 3 units

Course Pre-Requisite: Topology

Course Outline:

1. Review of Basic Topological Notions
   - 1.1. Connectedness, contractibility, convexity
   - 1.2. Simplexes
   - 1.3. Affine spaces and maps

2. Homotopy
    2.1. Fundamental groups
    2.2. Covering spaces

3. Singular Homology
    3.1. Singular homology and singular complex
    3.2. Homology functors
    3.3. Dimension and homotopy axioms
    3.4. Homology sequences
    3.5. Excision
    3.6. Mayer-Vietoris theorem

4. Simplicial Complexes
    4.1. Simplicial approximation
    4.2. Simplicial homology
    4.3. Calculations

5. Degree and Fixed Point Theorems
    5.1. Degrees of maps of spheres
    5.2. Fixed-point theorems
    5.3. Borsuk-Ulam theorems
    5.4. Applications

Suggested References:

1. A. Hatcher, *Algebraic Topology*, Cambridge Univ. Press, 2002.
2. J. Rotman, *An Introduction to Algebraic Topology*, Graduate Texts in Mathematics 119, Springer-Verlag, 1988.
3. S.P. Novikov, ed., *Topology I: General Survey*, Springer-Verlag, 1996.
4. J.R. Munkres, *Elements of Algebraic Topology*, Perseus Publ., 1984.
5. J.R. Munkres, *Topology,* Prentice-Hall

APPROXIMATION THEORY

Course Description:

Taylor's theorem, Weierstrass approximation theorem, approximation in Hilbert spaces, Fourier series and Fourier transform, direct and inverse theorems, algebraic and trigonometric interpolation, Whittaker-Shannon sampling theory, wavelet analysis.

Course Credit: 3 units

Course Pre-Requisite: Real Analysis I

Course Outline:

1. Taylor's Theorem and Weisertrass' Approximation Theorem
    1.1. Approximation of special functions and integrals
    1.2. Bernstein's polynomials

2. Approximation in Hilbert Spaces
    2.1. Orthogonal projections
    2.2. Orthonormal bases and Parseval's equality

3. Fourier Series and Fourier Integral Transform
    3.1. Jordan-Dirichlet Test and Fejer Sums
    3.2. Poisson summation formula and Shannon sampling theory

4. Wavelet Analysis
    4.1. Multiresolution analysis
    4.2. Wavelet Construction
    4.3. Convergence in various function spaces
    4.4. Frames

5. Direct and Inverse Theorems of Approximation
    5.1. Bernstein and Markov theorems
    5.2. Direct and inverse theorems of approximation

6. Interpolation
    6.1. Algebraic and trigonometric interpolation
    6.2. Convergence and non -convergence of interpolations

Suggested References:

1. O. Christensen and K. Christensen, *Approximation Theory: From Taylor Polynomials to Wavelets*, Boston Birkhauser, 2004.
2. G.G. Lorentz, *Approximation of Functions*, Chelsea Publishing Company, New York, N.Y., 1986.
3. R.A. De Vore, G .G. Lorentz, *Constructive Approximation*, Springer, Berlin, 1993.
4. I. Daubechies, *Ten Lectures in Wavelets*, American Math. Soc., 1992.

<u>CODING THEORY</u>

Course Description:

Fundamentals of the theory of error-correcting codes.

Course Credit: 3 units

Course Prerequisites: Undergraduate Abstract Algebra II, Linear Algebra

Course Outline:

1. Introductory Concepts
    1.1. Block codes, encoding and decoding, maximum -likelihood decoding
    1.2. Minimum-distance decoding
    1.3. Error detection and correction
    1.4. Shannon's noisy channel coding theorem

2. Linear Codes
    2.1. Minimum distance, generator and parity-check matrices, dual codes, standard array decoding, syndrome decoding
    2.2. Repetition codes, Hamming codes

3. Bounds on Code Parameters
    3.1. Hamming bound, Singleton bound
    3.2. Gilbert-Varshamov bound, Plotkin bound
    3.3. Using bounds to design good codes for a given set of parameters

4. Basic Finite Field Theory
    4.1. Definitions, prime fields
    4.2. Construction of prime power fields via irreducible polynomials, existence of primitive elements, minimal polynomials

5. Algebraic Codes
    5.1. Bose-Choudhury-Hocquenghem (BCH) and Reed -Solomon Codes
    5.2. Decoding of generalized Reed -Solomon codes
    5.3. Applications of Reed -Solomon codes in digital communications & storage
    5.4. Cyclic codes as ideals of polynomial rings
    5.5. Self-dual codes
    5.6. MacWilliams identity

6. Other topics to be selected, as time permits, such as:
    6.1. List decoding of Reed -Solomon codes
    6.2. Golay Codes
    6.3. Reed-Muller codes
    6.4. Codes over finite rings
    6.5. Goppa codes and algebraic geometry codes
    6.6. Convolutional codes, turbo codes
    6.7. Codes, expander codes, low -density parity-check (LDPC) codes

Suggested References:

1. W.C. Huffman and V. Pless, *Fundamentals of Error Correcting Codes*, Cambridge University Press, 2003.
2. J.H. van Lint, *Introduction to Coding Theory*, 3rd ed., Springer–Verlag (Graduate Texts in Mathematics series), 1999.
3. F.J. MacWilliams and N.J.A. Sloane, *The Theory of Error -Correcting Codes*, Elsevier/North -Holland, 197

COMBINATORIAL MATHEMATICS

Course Description:

Combinatorial principles and techniques, enumeration, combinatorial identities, combinatorial algorithms, topics from graph theory, sets and designs, algebraic and probabilistic principles and methods.

Course Credit: 3 units

Course Outline:

1. Enumeration
   1.1. Bijective arguments
   1.2. Generating functions
   1.3. Recurrence relations
   1.4. Inclusion-exclusion principle
   1.5. Bumside/Polya counting

2. Graphs
   2.1. Trees
   2.2. Circuits
   2.3. Matchings
   2.4. Connectivity
   2.5. Planarity

3. Sets
   3.1. Pigeonhole principle
   3.2. Ramsey's Theorem
   3.3. Turan's Theorem
   3.4. Dilworth's Theorem
   3.5. Sperner's Theorem

4. Methods
   4.1. Probabilistic combinatorics
   4.2. Methods from linear algebra

5. Designs
   5.1. Latin squares and projective planes
   5.2. Block designs
   5.3. Difference sets

6. Optimization
7. Perfect Graphs
8. Matroids

Suggested References:

1. Douglas West, *Combinatorics: A Core Course*, Upclose Printing, 2002.
2. Richard Stanley, *Enumerative Combinatorics I, II*

COMPLEX ANALYSIS

Course Description:

Functions of a complex variable, analytic functions, elementary complex functions, complex integration, complex series, singularities and residues, Maximum Modulus Theorem, analytic continuation, entire functions.

Course Credit: 3 units

Course Outline:

1. Functions of a Complex Variable
   1.1. Functions as Mappings
   1.2. Limits and Continuity
   1.3. Derivatives of Functions of a Complex Variable
   1.4. Analytic Functions and the Cauchy-Riemann Equations
   1.5. Harmonic Functions

2. Elementary Functions

3. Complex Integration
   3.1. Definitions and Fundamental Properties
   3.2. Contour Integration
   3.3. Cauchy Integral Theorem
   3.4. Cauchy Integral Formula

4. Complex Series
   4.1. Power Series and Regions of Convergence
   4.2. Taylor, Maclaurin and Laurent Series
   4.3. Algebraic Operations on Series
   4.4. Differentiation and Integration of Power Series

5. Theory of Residues
   5.1. Singularities
   5.2. Residues
   5.3. The Residue Theorem
   5.4. Applications to Improper Integrals and Other Special Classes of Integrals

6. Maximum Modulus Theorem
7. Compactness and Convergence in the Space of Analytic Functions
8. Runge's Theorem
9. Analytic Continuation
10. Harmonic and Entire Functions

Suggested References:

1. John B. Conway, *Functions of One Complex Variable*, Springer
2. L.V. Ahlfors, *Complex Analysis*
3. W. Rudin, *Real and Complex Analysis*
4. L.L. Pennisi, *Elements of Complex Variables*

DESIGN THEORY

Course Description:

Designs and their properties, existence theorems, constructions. Connections with graphs, finite geometries and codes.

Course Credit: 3 units

Course Outline:

1. Basic Concepts
   1.1. Basic Definitions and Properties
   1.2. Related Structures
   1.3. The Incidence Matrix
   1.4. Block's Lemma and the Orbit Theorem

2. Symmetric Designs
   2.1. Residual Structures
   2.2. The Bruck-Ryser-Chowla Theorem
   2.3. Singer Groups and Difference Sets
   2.4. Arithmetic Relations and Hadamard -2 Designs
   2.5. The Dembowski-Wagner Theorem

3. Some Families of Symmetric Designs
   3.1. Projective and Affine Planes
   3.2. Latin Squares
   3.3. Hadamard Matrices and Hadamard -2 Designs
   3.4. Biplanes
   3.5. Strongly Regular Graphs

4. Cameron's Theorem and Hadamard-3 Designs
5. Resolutions
6. Codes and Designs

Suggested References:

1. D.R. Hughes and F.C. Piper, *Design Theory*
2. P. Cameron and J.H. Van Lint, *Design, Graphs, Codes and Their Links*
3. M. Hall, *Combinatorial Mathematics*
4. A. Street and D. Street *Combinatorics and Experimental Design*

DIFFERENTIAL GEOMETRY

Course Description:

First course in differentiable manifolds and global analysis, basic tools in differential geometry and global analysis, applications of differential geometric methods.

Course Credit: 3 units

Course Pre-Requisite: Topology

Course Outline:

1. Manifolds
   1.1. Definition and examples of differentiable manifolds, partitions of unity

   1.2. Differentiable functions and mappings, implicit function theorem and rank theorem
   1.3. Immersions and embeddings, submanifolds
   1.4. Tangent vectors, tangent bundles

2. Calculus on Manifolds
   2.1. Vector fields, integral curves and flows, Lie bracket
   2.2. Differential forms, exterior calculus and algebra of differential forms.
   2.3. Closed and exact forms. Poincare Lemma. De Rham complex.
   2.4. Oriented manifolds, equivalent definitions of orientability
   2.5. Integration of differential forms. Stokes theorem

3. Connections
   3.1. Connections on vector bundles
   3.2. Parallel transport
   3.3. Levi-Civita connections

Suggested References:

1. M. Do Carmo, *Differential Geometry of Curves and Surfaces*
2. J. Oprea, *Differential Geometry and its Applications*, MAA, 2007.
3. A. Gray, E. Abbena and S. Salomon, *Modern Differential Geometry of Curves and Surfaces with Mathematica*, Chapman and Hall, CRC, 2007.
4. S. Kobayashi, *Foundations of Differential Geometry*, Wiley
5. F.W. Warner, *Foundations of Differentiable Manifolds and Lie Groups*, GTM, Springer

FUNCTIONAL ANALYSIS

Course Description:

Metric spaces, normed and inner product spaces, compact, self-adjoint operators, the Hahn -Banach Theorem and the Fixed-Point Theorem, uniform boundedness, approximation theory and the spectral theory of linear operators.

Course Credit: 3 units

Course Pre-Requisites: Real Analysis I, Linear Algebra

Course Outline:

1. Metric Spaces
   1.1. Metric Spaces
   1.2. Convergence and Completeness
   1.3. Completion of Metric Spaces

2. Normed Spaces
   2.1. Normed Spaces and Their Properties
   2.2. Banach Spaces
   2.3. Finite Dimensional Normed Spaces
   2.4. Linear Operators

- 2.5. Bounded Linear Operators
- 2.6. Linear Functionals
- 2.7. Dual Spaces

3. Inner Product Spaces
   - 3.1. Inner Product Spaces and Their Properties
   - 3.2. Hilbert Spaces
   - 3.3. Orthonormal Sets and Sequences
   - 3.4. Representations of Linear Functionals in Hilbert Spaces
   - 3.5. Hilbert-Adjoint Operator
   - 3.6. Self-Adjoint, Unitary and Normal Operators

4. Fundamental Theorems for Normed Spaces
   - 4.1. The Hahn-Banach Theorem
   - 4.2. Adjoint Operators
   - 4.3. The Uniform Boundedness Theorem
   - 4.4. Strong and Weak Convergence
   - 4.5. The Open Mapping Theorem
   - 4.6. Closed Linear Operators and the Closed Graph Theorem

5. Applications
   - 5.1. Banach Fixed Point Theorem
   - 5.2. Applications to Differential and Integral Equations
   - 5.3. Uniform Approximation
   - 5.4. Approximations in Hilbert Spaces

6. Spectral Theory of Linear Operators
   - 6.1. Spectral Theory in Finite Dimensional Normed Spaces
   - 6.2. Spectral Properties of Bounded Linear Operators

7. Spectral Theory of Compact Linear Operators
   - 7.1. Compact Linear Operators In Normed Spaces
   - 7.2. Spectral Properties of Compact Linear Operators

8. Spectral Theory of Bounded Self-Adjoint Linear Operators
   - 8.1. Spectral Properties of Bounded Self-Adjoint Linear Operators
   - 8.2. Positive Operators
   - 8.3. Projection Operators
   - 8.4. Spectral Family

Suggested References:

1. J. Conway, *A Course in Functional Analysis*
2. W. Rudin, *Functional Analysis*
3. E. Kreysig, *Introductory Functional Analysis with Applications*

GEOMETRIC CRYSTALLOGRAPHY

Course Description:

Isometries, frieze groups, crystallographic groups, lattices and invariant sublattices, finite groups of isometries, geometric and arithmetic crystal classes.

Course Credit: 3 units

Course Pre-Requisite: Abstract Algebra I

Course Outline:

1. Isometries and their products	(1.5 weeks)
2. Cyclic groups, dihedral groups	(1 week)
3. Frieze groups	(2 weeks)
4. The 17 plane crystallographic groups	(3 weeks)
5. Euclidean Lattices	(1.5 weeks)
6. Cohomology and the derivation of the 17 plane groups	(2 weeks)
7. Subgroup structures of the 17 plane groups	(2 weeks)
8. Finite groups of isometries in dimension 3	(1.5 weeks)
9. Bravais lattices	(1.5 weeks)

Suggested References:

1. Brown, et al., *Crystallographic Groups of Four-Dimensional Space*, John Wiley & Sons, Inc., 1978.
2. P. Engel, *Geometric Crystallography*, Reidel Publ., 1986.
3. Hahn, ed. *International Tables for Crystallography*, Reidel Publ., 2nd ed. 1988.
4. R.L.E. Schwarzenberger, *N-dimensional Crystallography*, Pitman, 1980.
5. M. Senechal, *Crystalline Symmetries*, Adam Hilger, 1990.

GRAPH THEORY

Course Description:

Fundamental concepts in graph theory. Graphs, subgraphs, adjacency, connectedness, vertex-regularity, and chromatic number. Special classes of graphs. Applications.

Course Credit: 3 units

Course Outline:

1. Fundamental Concepts
   - 1.1. Vertices, Edges, Order and Size
   - 1.2. Paths, Walks and Cycles
   - 1.3. Subgraphs: Proper, Induced and Spanning

2. Some Special Classes of Graphs
   - 2.1. Path, Wheel and Cycle Graphs
   - 2.2. Complete, Bipartite and Multipartite Graphs
   - 2.3. Connected Graphs and Graph Components

2.4. Regular Graphs
2.5. Graph Isomorphisms

3. Trees
   3.1. Basic Properties
   3.2. Distance in Trees and Graphs
   3.3. Shortest Paths and Spanning Trees
   3.4. Graceful Tree Conjecture

4. Traversability
   4.1. Hamiltonian Graphs
   4.2. Eulerian Graphs
   4.3. Applications
   4.4. Chinese Postman Problem
   4.5. Traveling Salesman Problem

5. Digraphs
   5.1. Digraph Invariants
   5.2. Tournaments

6. Planarity
   6.1. Planar Graphs
   6.2. Fundamental Results on Planar Graphs: Euler's Theorem, Kuratowski's Theorem
7. Graph Coloring
   7.1. Vertex Coloring
   7.2. Chromatic Number
   7.3. Four-Color Theorem
   7.4. Perfect Graphs

8. Matching and Factors
   8.1. Matchings
   8.2. Graph Factorization
   8.3. Graph Decomposition

Suggested References:

1. F. Harary, *Theory of Graphs*
2. J.A. Bondy and U.S.R. Murty, *Graph Theory with Applications*
3. G. Chartrand and L. Lesniak, *Graphs and Digraphs*
4. D.B. West, *Introduction to Graph Theory*

GROUP THEORY

Course Description:

Advanced topics in the theory of groups.

Course Credit: 3 units

Course Pre-Requisite: Abstract Algebra I

Course Outline:

1. Automorphisms of groups
2. p-groups
3. Linear groups, symmetric and alternating groups
4. Permutation groups, multiply-transitive groups
5. Semi-direct products and wreath products
6. Subnormal series,
7. Solvable groups
8. Nilpotent groups
9. Central series, composition series, Jordan-Hölder Theorem
10. Extensions of groups
11. Some simple groups

Suggested References:

1. J. Rotman, *An Introduction to the Theory of Groups, Graduate Texts in Mathematics 148*, 4th Ed., Springer, 1995.
2. M. Suzuki, *Group Theory I and II*, Grundlehren der Mathematischen Wissenschaften, vol. 247-248, Springer-Verlag, 1982 and 1986.
3. J. Humphreys, *A Course in Group Theory*, Oxford University Press, 1996.
4. Rose, *A Course on Group Theory*, Cambridge University Press, 1978 (reprinted by Dover, 1994).

HYPERBOLIC GEOMETRY

Course Description:

Moebius transformations, hyperbolic plane and hyperbolic metric, geometry of geodesics, hyperbolic trigonometry, groups of isometries on the hyperbolic plane.

Course Credit: 3 units

Course Outlines:

1. Moebius transformations and the Moebius group (2 weeks)
2. The hyperbolic plane and hyperbolic metric (3weeks)
3. Geometry of geodesics (2 weeks)
4. Hyperbolic trigonometry (2 weeks)
5. Isometries of the hyperbolic plane (3 weeks)
6. Triangle groups (2 weeks)
7. Fuchsian groups (2 weeks)

Suggested References:

1. A. Beardon, *An Introduction to Hyperbolic Geometry*
2. T. Bedford, M. Keane, *Ergodic Theory, Symbolic Dynamics and Hyperbolic Spaces*, Oxford University Press, 1991.
3. A. Beardon, *The Geometry of Discrete Groups*, Springer-Verlag, 1983.
4. R. C. Lyndon, *Groups and Geometry*, London Mathematical Society Lecture Note Series 101, Cambridge University Press, 1985.
5. W. Magnus, *Non-euclidean Tesselations and their Groups*, Academic Press, 1974.

LIE ALGEBRAS AND LIE GROUPS

Course Description:

Introduction to Lie groups, Lie algebras of Lie groups, nilpotent and solvable algebras, semi-simple Lie algebras, representations.

Course Credit: 3 units

Course Pre-Requisite: Modern Algebra I

Course Outline:

1. Basic Definition and Examples
   - 1.1. Subalgebras, ideals, field extensions, homomorphisms
   - 1.2. Associative algebras, matrix algebras, low -dimensional examples, algebras of differential operators
   - 1.3. Introduction to Lie groups

2. Nilpotent and Solvable Lie Algebras
   - 2.1. Definitions and examples; Lie's Theorem, Engel's Theorem
   - 2.2. Semi-simple Lie algebras, Cartan's criterion for semi-simplicity
   - 2.3. Nilpotent, solvable Lie groups, classical semi-simple Lie groups

3. Complex Semi-Simple Lie Algebras
   - 3.1. Classical root space decompositions
   - 3.2. Cartan subalgebras, Weyl group
   - 3.3. Classification of Cartan matrices
   - 3.4. Root systems and the Serre relations

4. Compact Lie Groups
   - 4.1. Introduction to representation theory of Lie groups and Lie algebras
   - 4.2. Examples, sl(2, C), Peter-Weyl Theorem
   - 4.3. Compact Lie algebras, analytic Weyl group, integral forms

Suggested References:

1. Knapp, *Lie Groups Beyond an Introduction 2nd ed.*, Birkhauser, 2001.
2. Humphreys, *Introduction to Lie Algebras and Representation Theory,* Graduate Texts in Mathematics, Springer-Verlag, 1987.
3. J.P. Serre, *Complex Semi-simple Lie Algebras*, Springer-Verlag, 1987.

4. W. Fulton and J. Harris, *Representation Theory: A First Course,* Graduate Texts in Mathematics, Springer-Verlag, 1987.

LINEAR ALGEBRA

Course Description:

Vector spaces, linear transformations and matrices, determinants, eigenvalues, canonical forms, bilinear forms and quadratic forms.

Course Credit: 3 units

Course Pre-requisite: Undergraduate Linear Algebra

Course Outline:

1. Vector Spaces
   - 1.1. Basic Concepts
   - 1.2. Linear Independence
   - 1.3. Bases
   - 1.4. Subspaces

2. Linear Transformations and Matrices
   - 2.1. Linear Transformations
   - 2.2. Matrices
   - 2.3. Elementary Matrices
   - 2.4. Row Equivalence
   - 2.5. Change of Bases
   - 2.6. Normal Forms

3. Determinants, Eigenvalues and Similarity Transformations
   - 3.1. Determinants
   - 3.2. Eigenvalues and Eigenvectors
   - 3.3. Diagonalization
   - 3.4. Hamilton-Cayley Theorem
   - 3.5. Minimal Polynomials
   - 3.6. Invariant Subspaces
   - 3.7. Jordan Canonical Form

4. Linear Functionals, Bilinear Forms, and Quadratic Forms
   - 4.1. Linear Functionals
   - 4.2. Annihilators
   - 4.3. Bilinear Forms
   - 4.4. Quadratic Forms
   - 4.5. Hermitian Forms

5. Inner Product Spaces
   - 5.1. Inner Products and Norms
   - 5.2. The Adjoint of a Linear Operator
   - 5.3. Normal and Self -Adjoint Operators

5.4. Unitary and Orthogonal Operators

Suggested References:

1. D. T. Finkbeiner , *Elements of Linear Algebra*
2. T.W. Hungerford, *Algebra*, Springer.
3. E.D. Nering, *Linear Algebra and Matrix Theory*
4. S. Lang, *Linear Algebra*

MATHEMATICS IN POPULATION BIOLOGY

Course Description:

Continuous population models for single species, discrete population models for single species, models for interacting populations, evolutionary models. Dynamics of infectious diseases.

Course Credit: 3 units

Course Pre-Requisite: Ordinary Differential Equations

Course Outline:

1. Continuous Population Models for Single Species (3 weeks)
   - 1.1. Continuous growth models
   - 1.2. Delay population models
   - 1.3. Population models with age distribution
   - 1.4. Single species, bifurcation, and stability

2. Discrete Population Models for Single Species (3 weeks)
   - 2.1. Cobwebbing
   - 2.2. Discrete logistic-type models
   - 2.3. Discrete delay models

3. Models of Interacting Populations (3 weeks)
   - 3.1. Predator-prey models: Lotka-Volterra systems
   - 3.2. Analysis of realistic predator-prey models
   - 3.3. Competition models
   - 3.4. Discrete growth models for interacting populations

4. Epidemiology Models (3 weeks)
   - 4.1. Simple epidemic models
   - 4.2. Modeling venereal diseases
   - 4.3. Modeling the transmission dynamics of HIV

5. Dynamics of Infectious Diseases (3 weeks)
   - 5.1. Delay model for HIV infection with drug therapy
   - 5.2. Age-dependent epidemic model
   - 5.3. Simple drug use epidemic model

Suggested References:

1. N.F. Britton, *Essential Mathematical Biology*, Springer, 2003.
2. Leah Edelstein-Keshet, *Mathematical Models in Biology*, SIAM, 2005.
3. Alan Hastings, *Population Biology: Concepts and Models*, Springer, 1998.
4. M. Kot. *Elements of Mathematical Ecology*. Cambridge University Press, 2001.
5. J.D. Murray, *Mathematical Biology I. An Introduction*, Springer, 2001.

MATHEMATICAL FINANCE

Course Description:

Binomial asset pricing model, vanilla options, exotic options, American options, arbitrage probabilities, profit and loss, stochastic interest rates.

Course Credit: 3 units

Course Outline:

1. Binomial Asset Pricing Model (4 weeks)
   - 1.1. Binomial model for stock price dynamics
   - 1.2. Review of conditional expectation
   - 1.3. Martingales

2. Pricing Exotic Options (3 weeks)
   - 2.1. Hedging
   - 2.2. Reflection principle for Brownian motion
   - 2.3. Up and out European call

3. Asian Options (3 weeks)
   - 3.1. Feynman-Kac Theorem
   - 3.2. Constructing the hedge
   - 3.3. Partial average payoff Asian option

4. Arbitrage Pricing Model (3 weeks)
   - 4.1. Binomial model, hedging portfolio
   - 4.2. Setting up the continuous modeling
   - 4.3. Risk-neutral pricing and hedging
   - 4.4. Implementation of risk-neutral pricing and hedging

5. American Put Options (3 weeks)
   - 5.1. Value of perpetual American put
   - 5.2. Hedging the put
   - 5.3. Perpetual American call
   - 5.4. American put contingent with expiration

Suggested References:

1. Pliska, S. *Introduction to Mathematical Finance*. Blackwell Publishers, Oxford UK, 2001.
2. Shreve, S. *Stochastic Calculus for Finance Vol. I: The Binomial Asset Pricing*

Model. Springer-Verlag, New York, 2004.
3. Shreve, S. *Stochastic Calculus for Finance Vol. II: Continuous -Time Models.* Springer-Verlag, New York, 2004.

MATHEMATICAL STATISTICS

Course Description:

Probability spaces, random variables, distribution functions, independence of random variables, expectation, convergence, characteristic functions, strong law of large numbers and central limit theorem.

Course Credit: 3 units

Course Outline:

1. Probability Spaces
    1.1. Random Experiments and Sample Spaces
    1.2. Events and Classes of Sets
    1.3. Probabilities and Probability Spaces
    1.4. Probabilities on
    1.5. Conditional Probability

2. Random Variables
    2.1. Fundamentals of Random Variables
    2.2. Distributions and Distribution Functions
    2.3. Density and Mass Functions
    2.4. Common Families of Distributions
    2.5. Exponential, Location and Scale Families
    2.6. Transformation Theory: Distributions of Functions of Random Variables

3. Independence
    3.1. Independent Random Variables
    3.2. Functions of Independent Random Variables
    3.3. Independent Events

4. Expectation
    4.1. Basic Properties
    4.2. Integrals with respect to Distribution Functions
    4.3. Computation of Expectations
    4.4. Lp Spaces and Inequalities
    4.5. Moments

5. Convergence of Sequences of Random Variables
    5.1. Modes of Convergence
    5.2. Relationships Among the Modes of Convergence
    5.3. Convergence under Transformations

6. Characteristic Functions
    6.1. Basic Properties

6.2. Inversion and Uniqueness Theorems
6.3. Moments and Taylor Expansions
6.4. Continuity Theorems and Applications
6.5. Other Transforms

7. Laws of Large Numbers
   7.1. The Strong Law of Large Numbers
   7.2. The Central Limit Theorem

Suggested References:

1. Bhat, B.R. (2009). *Modern Probability Theory, An Introductory Textbook*, 3/e. New Age International.
2. Billingsley, P. (1995). *Probability and Measure*, 3/e. John Wiley & Sons, NY.
3. Casella, G. and Berger, R. (2002). *Statistical Inference*, 2/e. Duxbury, CA.
4. Karr, Allan F. (1993). *Probability*. Springer-Verlag, New York.
5. Rao, M.M. and Swift, R.J. (2006). *Probability Theory with Applications*, 2/e. Springer-Verlag, New York.
6. Shorack, G.R. (2000). *Probability for Statisticians*. Springer-Verlag, New York.

MULTIVARIATE ANALYSIS

Course Description:

Principal components analysis, factor analysis, cluster analysis, discriminant and classification, linear and logistic regression, neural networks, applications.

Course Credit: 3 units

Course Pre-Requisites: Mathematical Statistics, Linear Algebra

Course Outline:

1. Principal Components Analysis
   1.1. Inertia and centroid of a cloud
   1.2. First factorial and other factorial axes
   1.3. Procedure of a principal components analysis
   1.4. Principal components in the space of variables
   1.5. The correlation disk

2. Factor Analysis
   2.1. The orthogonal factor model
   2.2. Methods of estimation
   2.3. Factor rotation and factor scores
   2.4. Procedure for a factor analysis

3. Cluster Analysis
   3.1. Distance matrix
   3.2. Huygen's formula
   3.3. Hierarchal clustering methods

3.4. K-means method

4. Discriminant and Classification
    4.1. Decomposition of the covariance matrix
    4.2. Discriminant using Mahalanobis distance
    4.3. Discriminant vectors
    4.4. Fisher's method of discriminating among several samples

5. Linear and Logistic Regression
    5.1. Linear regression model
    5.2. Logistic regression

6. Introduction to Neural Net Methods
    6.1. Multi-layers perception
    6.2. Traditional methods with the point of view of neural net
    6.3. Kohenen algorithms (self-organizing maps)

7. Examples and Applications

Suggested References:

1. Hastie, Tibshirani and Friedman, *The Elements of Statistical Learning*, Springer, 2001.
2. Johnson and Wichern, *Applied Multivariate Statistical Analysis, 5th Ed.*, Prentice Hall, 2002.
3. Manly, B., *Multivariate Statistical Methods, a primer, 3rd Ed.*, Chapman and Hall, 2005.

NUMERICAL ANALYSIS I

Course Description:

Introductory course in numerical analysis that shows how numerical methods are used to solve various computational problems, such as solutions of nonlinear equations, using interpolating polynomials to approximate derivatives and integrals, solving systems of linear equations, the eigenvalue problem and numerical solutions to differential equations. The mathematical bases for these numerical methods are likewise presented.

Course Credit: 3 units

Course Outline:

1. Errors and Approximations
    1.1. The Taylor Polynomial
    1.2. Floating Point Arithmetic
    1.3. Errors: Types and Sources
    1.4. Error Propagation

2. Solutions of Non-Linear Equations
    2.1. Bisection Method

   - 2.2. Method of False Position
   - 2.3. Newton's Method
   - 2.4. Secant Method
   - 2.5. Fixed Point Iteration

3. Interpolating Polynomials
   - 3.1. Polynomial Interpolation
   - 3.2. Forward, Backward and Central Difference Formulas
   - 3.3. Error of Polynomial Interpolation
   - 3.4. Interpolation Using Cubic and other Spline Functions
   - 3.5. Chebyshev Polynomials
   - 3.6. Least Squares Approximation

4. Numerical Integration and Differentiation
   - 4.1. Numerical Differentiation Using Interpolation
   - 4.2. The Trapezoidal and Simpson's Rules
   - 4.3. Gaussian Quadratures
   - 4.4. Romberg Integrals

5. Solutions of Linear Systems
   - 5.1. Gaussian Elimination
   - 5.2. LU-Decomposition
   - 5.3. Iterative Methods: The Gauss-Jacobi and Gauss-Seidel methods
   - 5.4. Error Analysis
   - 5.5. Residual Correction
   - 5.6. Stability

6. The Eigenvalue Problem

7. Numerical Solutions of Differential Equations
   - 7.1. Euler's Method
   - 7.2. Richardson Extrapolation
   - 7.3. Runge-Kutta Methods
   - 7.4. Multistep methods
   - 7.5. Finite Difference Methods for Partial Differential Equations

Suggested References:

1. K. Atkinson and W. Han, *Elementary Numerical Analysis*
2. R. Burden and D. Faires, *Numerical Analysis*
3. A. Ralston and P. Rabinowitz, *A First Course in Numerical Analysis*

NUMERICAL ANALYSIS II

Course Description:

Numerical methods for ordinary differential equations, finite-difference methods for partial differential equations, numerical methods for conservation laws, multi-grid methods.

Course Credit: 3 units

Course Pre-Requisite: Numerical Analysis I

Course Outline:

1. Numerical Methods for Ordinary Differential Equations (4 weeks)
   1.1. The mathematical problem, existence and uniqueness
   1.2. Single step methods, implicit methods
   1.3. Convergence and stability
   1.4. Error estimation and control
   1.5. Stiffness

2. Finite-Difference Methods for Partial Differential Equations (6 weeks)
   2.1. Parabolic problems
   2.2. Elliptic problems

3. Numerical Methods for Conservation Laws (4 weeks)
   3.1. The Riemann problem, weak solutions, entropy conditions
   3.2. Scalar hyperbolic equations and the method of characteristics

4. Multigrid Methods (2 weeks)

Suggested References:

1. W. Briggs, V. Henson and S. Mccormick, *A Multigrid Tutorial*, SIAM, 2000.
2. R. LeVeque, *Numerical Methods for Conservation Laws*, Birkhauser Verlag, 1990.
3. L. Shampine, *Numerical Solution of Ordinary Differential Equations*, CRC Press, 1994.
4. J. Stoer and R. Bulisrch, *Introduction to Numerical Analysis*, 2nd Edition, Springer-Verlag, 1993.

NUMERICAL OPTIMIZATION

Course Description:

Deterministic descent type methods, stochastic optimization methods, numerical implementation.

Course Credit: 3 units

Course Pre-Requisite: Numerical Analysis I

Course Outline:

1. Introduction and General Theorems (2 weeks)
   1.1. Global and local optimization with or without constraints
   1.2. Conditions for existence and uniqueness
   1.3. The case of convex and linear functions

2. Deterministic Descent Type Methods (4 weeks)
   2.1. First and second order descent type methods

   2.2. Linear search principles
   2.3. Exact or approximate sensitivity computation
   2.4. Numerical implementation

3. Stochastic Optimization Methods (4 weeks)
   3.1. Simulated annealing
   3.2. Genetic algorithms
   3.3. Multi-objective programming
   3.4. Numerical implementation

4. Other Optimization Methods (3 weeks)
   4.1. Hybrid methods
   4.2. Linear programming
   4.3. Numerical implementation

Suggested References:

1. J.F. Bonnans, J.C. Gilbert, C. Lemarechal, C. Sagastizbal, *Numerical Optimization, Theoretical and Practical Aspects*, Springer, 2003.
2. J. Nocedal, S.J. Wright, *Numerical Optimization*, Springer, 1999.
3. Goldberg, *Genetic Algorithms on Search, Optimization and MachineLearning*, Cambridge University Press, 1989.

PROBABILITY THEORY

Course Description:

Random variables, special probability distributions, laws of large numbers, central limit theorem, Markov chains, Poisson process, martingales.

Course Credit: 3 units

Course Pre-Requisite: Real Analysis I

Course Outline:

1. Probability Measures (3 weeks)
   5.1. Probability spaces and sigma-field
   5.2. Random variables, expectation, independence
   5.3. Special probability distributions
   5.4. Borel-Cantelli Lemmas, Kolmogorov's zero -one law

6. Sums of Independent Random Variables (2 weeks)
   6.1. Laws of large numbers
   6.2. Maximal inequalities and random series

7. Markov Chains (2 weeks)
   7.1. Transience and persistence
   7.2. Optimal stopping

8. The Poisson Process (3 weeks)

8.1. Exponential distribution
8.2. Poisson approximation

9. Central Limit Theory (3 weeks)
   9.1. Characteristic functions
   9.2. Inversion and uniqueness theorems
   9.3. Lindeberg and Lyapunov theorems

10. Martingales (3 weeks)
    10.1. Radon-Nikodym Theorem
    10.2. Conditional expectation
    10.3. Martingales, gambling, stopping times
    10.4. Convergence theorems

Suggested References:

1. P. Billingsley, *Probability and Measure*, John Wiley and Sons, 1995.
2. K.L. Chung, *A Course in Probability Theory*, Academic Press, San Diego CA, 2001.
3. W. Feller, *An Introduction to Probability Theory and Its Applications*, Vol. I and II, Wiley, New York, NY, 1971.

PROJECTIVE GEOMETRY

Course Description:

Affine spaces. Projective spaces, linear subspaces, homogeneous coordinates, projective transformations and general position. Desargues' and Pappus theorems. Duality. Dual projective spaces. Harmonic sequences and Cross ratio. Projective and affine groups. Finite projective geometries.

Course Credit: 3 units

Course Outline:

1. The Projective Plane
2. Projective Spaces and Homogeneous Coordinates
3. Desarguesian planes
4. Perspectivity and Projectivity
5. Pappus Theorem
6. Analytic geometry and matrix representations
7. Projective collineations
8. Fundamental Theorem of Projective Geometry
9. Harmonic Sequences and Cross Ratio
10. The General Linear Group
11. Projective and Affine Groups
12. Finite Projective Geometries

Suggested References:

1. H.S.M. Coxeter, *Projective Geometry*
2. Bennett, *Affine and Projective Geometry*

3. D.R. Hughes and F.C. Piper, *Projective Planes*
4. R. Casse, *Projective Geometry: An Introduction*, Oxford, 2006.

REAL ANALYSIS I

Course Description:

Measure theory and Lebesgue integration. Outer measure, measurable sets, Lebesgue measure, the Lebesgue integral, differentiation, integration, and L^p spaces, Hilbert spaces and Fourier series.

Course Credit: 3 units

Course Pre-Requisites: Undergraduate Advanced Calculus or Real Analysis

Course Outline:

1. Review of Real Number System
   - 1.1. Rationals, completeness
   - 1.2. Topology of the real line, Bolzano-Weierstrass theorem, Heine-Borel theorem Connectedness
   - 1.3. Review of Continuous Functions
   - 1.4. Basic definitions and properties
   - 1.5. Uniform continuity
   - 1.6. Weierstrass approximation theorem

2. Review of Differentiality

3. Review of Riemann Integral
   - 3.1. Basic definitions and properties
   - 3.2. Integrability of continuous functions
   - 3.3. Differentiability of integral at points of continuity

4. The Elementary Functions and Their Basic Properties

5. Sequences and Series
   - 5.1. Basic definitions and properties
   - 5.2. Cauchy criteria, limsup, liminf, sequences and series of functions
   - 5.3. Uniform convergence
   - 5.4. Power series, Taylor series

6. Lebesgue Measure on the Line
   - 6.1. Basic definitions and properties
   - 6.2. Lebesgue outer measure, Lebesgue measurability
   - 6.3. The Lebesgue theorem on Riemann integrability
   - 6.4. Cantor sets

7. Lebesgue Measurable Functions
   - 7.1. Basic definitions and properties
   - 7.2. Modes of convergence
   - 7.3. Egorov's theorem, Lusin's theorem

8. Lebesgue Integral
   8.1. Basic definitions and proper ties
   8.2. Convergence theorems

9. Differentiability
   9.1. Monotone functions
   9.2. Bounded variation
   9.3. Absolute continuity, differentiability of integral, integrability of derivative

10. L^p Spaces on Intervals
    10.1. Basic definitions and properties
    10.2. Fundamental inequalities
    10.3. Completeness

11. Fourier Series on an Interval
    11.1. Riemann-Lebesgue lemma
    11.2. Bessel's inequality, Parseval's formula
    11.3. Baire Category Theorem and elementary consequences

Suggested References:

1. H.L. Royden, *Real Analysis*, 3rd edition, Prentice Hall, 1988.
2. W. Rudin, *Real and Complex Analysis*

<u>REAL ANALYSIS II</u>

Course Description:

Further topics on L^p spaces, normed and Banach spaces, linear operators, fundamental theorems on normed spaces (Hahn-Banach, open mapping, closed graph, Banach-Steinhaus theorems, and uniform boundedness principle), Hilbert spaces, signed and complex measures, Fourier transform (inversion formula and Plancherel's theorem)

Course Credit: 3 units

Course Pre-Requisite: Real Analysis I

Course Outline:

1. Review of Hilbert Spaces
2. Banach Space techniques
3. Complex measures, the Radon-Nikodym theorem, bounded linear functional on L^p spaces
4. Integration on product spaces, Fubini's theorem
5. Derivatives of measures, Lebesgue points of an integrable function, differentiability of functions of bounded variation
6. The Fourier transform (inversion formula, Plancherel's Theorem)

Suggested References:

1. G. Folland, *Real Analysis*, John Wiley and Sons
2. W. Rudin, *Real and Complex Analysis*
3. E.M. Stein and R. Shakarchi, *Real Analysis*, Princeton University Press, 2005.

STOCHASTIC CALCULUS

Course Description:

Conditional expectations, martingales, Brownian motion, Ito integral, Ito formula, stochastic differential equation, Girsanov Theorem, applications to mathematical finance.

Course Credit: 3 units

Course Pre-Requisite: Mathematical Statistics

Course Outline:

1. Conditional Expectation (2 weeks)
   1.1. Conditioning against a random variable
   1.2. Properties of conditional expectation

2. Martingales (2 weeks)
   2.1. Information and filtration
   2.2. Optimal stopping time

3. The Bond-Stock Market (2 weeks)
   3.1. Portfolio and arbitrage
   3.2. Risk neutral probability
   3.3. Girsanov Theorem

4. Option Pricing (2 weeks)
   4.1. Pricing a European option
   4.2. Pricing an American option

5. Brownian Motion (2 weeks)
   5.1. Continuous time process
   5.2. Properties of Brownian motion
   5.3. Continuous time martingales

6. Ito Calculus (3 weeks)
   6.1. Ito integral and Ito's process
   6.2. Girsanov Formula

7. Black-Scholes Model (2 weeks)
   7.1. Portfolio in continuous time setting
   7.2. Risk neutral probability in continuous time setting
   7.3. Option pricing in the Black-Scholes model

Suggested References:

1. Baxter, M., Rennie A. Financial Calculus: *An introduction to Derivative Pricing*. Cambridge University Press, 1999.
2. Mel'nikov, A.V. *Financial Markets: Stochastic Analysis and the Pricing of Derivative Securities. Translations of Mathematical Monographs, AMS edition*, Providence, 1999.
3. Pliska, S. *Introduction to Mathematical Finance*. Blackwell Publishers, Oxford UK, 2001.
4. Williams, D. *Probability with Martingales. Cambridge Mathematical Textbooks*, Cambridge University Press, 1991.
5. Steven E. Shreve. *Stochastic Calculus for Finance (v. 1, II)*, 2004. Springer Finance.
6. Thomas Björk, *Arbitrage Theory in Continuous Time (2nd ed)*, 2004, Oxford University Press.

THEORY OF ORDINARY DIFFERENTIAL EQUATIONS

Course Description:

Existence and uniqueness theorems. Equilibrium and Linearization. Nonlinear equations. Hamiltonian systems. Bifurcation.

Course Credit: 3 units

Course Pre-Requisites: Undergraduate Ordinary Differential Equations

Course Outline:

1. Existence, Uniqueness
   - 1.1. Vector fields and flows
   - 1.2. Existence and uniqueness theorems
   - 1.3. Examples of non-uniqueness
   - 1.4. Continuation of solutions

2. Equilibrium and Linearization
   - 2.1. Equilibrium and stability
   - 2.2. Linearization about a solution and the equation of variation
   - 2.3. Solution of linear systems via exponential map
   - 2.4. Almost linear systems and linear stability
   - 2.5. Hyperbolic fixed points, stable and unstable manifolds
   - 2.6. Liapunov functions and Liapunov stability

3. Geometric Methods for Nonlinear Equations
   - 3.1. Limit sets and asymptotic behavior
   - 3.2. Phase portrait methods in 2 and 3 dimensions
   - 3.3. Periodic orbits and the Poincaré-Bendixon theorem

4. Hamiltonian Systems
   - 4.1. Hamiltonian flows
   - 4.2. First integrals

   4.3. Symplectic matrices
   4.4. Poincare maps

5. Bifurcation Theory
   5.1. Bifurcation of equilibria
   5.2. Hopf bifurcation theorem
   5.3. Lorenz system

6. Area Preserving Maps
   6.1. Area preserving maps
   6.2. Elliptic fixed points
   6.3. Poincaré-Birkhoff theorem

Suggested References:

1. Coddington and Levinson, *Theory of Ordinary Differential Equations*, Krieger Publishers, 1984.
2. Arowsmith and Place, *An Introduction to Dynamical Systems*, Cambridge Univ. Press, 1990.

THEORY OF PARTIAL DIFFERENTIAL EQUATIONS

Course Description:

First and second order equations. Sobolev and function spaces. Non -smooth solutions, variational techniques, fixed point theorems. Wave equation. Laplace's equation. Heat equation.

Course Credit: 3 units

Course Pre-Requisite: Real Analysis I

Course Outline:

1. Partial Differential Equations
   1.1. Definition and Examples
   1.2. Well-posed and Ill-posed Problems
   1.3. Boundary and Initial Value Conditions

2. Second Order PDEs
   2.1. Elliptic, Hyperbolic and Parabolic PDEs
   2.2. Canonical Forms

3. Classical Solutions
   3.1. Transport Equation
   3.2. Wave Equation
   3.3. Heat Equation
   3.4. Laplace Equation

4. Some Existence and Uniqueness Theorems

5. Introduction to Sobolev Spaces
    5.1. Weak Convergence in Banach Spaces
    5.2. The Lax-Milgram Theorem
    5.3. Variational Formulation Setting
    5.4. Some Variational Elliptic Problems (Dirichlet, Neumann, Robin)

Suggested References:

1. L.C. Evans, *Partial Differential Equations*
2. D. Cioranescu, P. Donato and M.P. Roque, *An Introduction to Partial Differential Equations and Sobolev Spaces*
3. D. Cioranescu, *An Introduction to Homogenization*
4. P. Duchateau and D.W. Zachmann, *Theory and Problems of Partial Differential Equations*

TOPOLOGY

Course Description:

Topological spaces, connectedness, product space, identification topology, separation axioms, metric spaces, uniform spaces, convergence, and filter bases.

Course Credit: 3 units

Course Outline:

1. Topological Spaces
    1.1. Definitions and examples
    1.2. Basis for a topology
    1.3. Order topology
    1.4. Product topology
    1.5. Subspace topology
    1.6. Closed sets and limit points
    1.7. Box topology
    1.8. Metric topology
    1.9. Quotient topology

2. Connectedness and Compactness
    2.1. Connected spaces, connected subspaces of the real line
    2.2. Components and local connectedness
    2.3. Compact spaces, compact subspaces of the real line
    2.4. Limit point compactness
    2.5. Local compactness

3. Countability and Separation Axioms
    3.1. Countability axioms
    3.2. Separation axioms
    3.3. Normal spaces
    3.4. Urysohn Lemma and Urysohn Metrization Theorem

4. Tychonoff Theorem and Stone-Cech Compactification

5. Complete Metric Spaces and Function Spaces
    5.1. Complete metric spaces
    5.2. Compactness in metric spaces
    5.3. Point-wise and compact convergence
    5.4. Ascoli's Theorem

Suggested References:

1. J.R. Munkres, *Topology: A First Course*, Prentice-Hall, 2nd ed.
2. J. Dugundji, *Topology*, Allyn & Bacon, Boston
3. K. Jänich, *Topology*, Springer